Steepener-Neutral $\iff$ Convexity-Neutral

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Tenure

The yield of a strippable bond must be the appropriately-weighted average of the yields of its strips. Let's find the weights, and then the time-centre of those weights.

Define that yields are quoted continuously compounded, that every instrument's settlement date is at time 0, and that strip i pays nominal **m**oney cashflow m_i at **t**ime t_i, and that its price per 1 payment is $disc_i$. Starting with a flat yield curve, so the bond and all its strips have the same yield, nudge the continuously-compounded yield of strip i by $+\varepsilon$. The change in strip price is $-\varepsilon\ t_i\ m_i\ disc_i$, so the price of the bond must change by the same amount. Hence the yield of the bond must rise by this divided by the bond's BPV, so must rise by $+\varepsilon\ t_i\ m_i\ disc_i\ /\ (\sum_j t_j\ m_j\ disc_j)$. So, at least with yields nearly flat, the continuously-compounded yield of a bond is, to first order, the weighted average of the continuously-compounded yields of its strips, with weights $\propto t_i\ m_i\ disc_i = \text{BPV}_i$.

The time-centre of these weights, the average location of the source of the yield, the natural x value for a bond, defined as 'tenure'[†] $= (\sum t_i^2\ m_i\ disc_i)\ /\ (\sum_i t_i\ m_i\ disc_i)$. Divide top and bottom by the bond's price, and this Macaulay-Weil tenure is Macaulay-Weil convexity ÷ Macaulay-Weil duration. There is also the obvious Weil-free variant, Macaulay tenure = Macaulay convexity ÷ Macaulay duration.

Steepener-Neutral

Define that, in a multi-bond trade, bond b has Macaulay duration dur_b, Macaulay convexity conv_b, and Macaulay tenure = $\text{tenure}_b = \text{conv}_b/\text{dur}_b$, and the money value owned is money_b.

It is required that the trade be steepener neutral. This is defined to mean that the trade is neutral to a linear steepening of continuously-compounded yields, where the x value of each bond is its tenure. That is, if, $\forall b$, the yield of bond b instantaneously changes by $\varepsilon \cdot \text{tenure}_b$, then there is neither profit nor loss

$$\iff \quad 0 \;=\; \sum\nolimits_b (\Delta(\text{yield}_b)\ \cdot\ \text{BPV}_b) \;=\; \sum\nolimits_b (\varepsilon\ \text{tenure}_b\ \cdot\ \text{dur}_b\ \text{money}_b)\ \ \forall\varepsilon$$

$$\iff \quad 0 \;=\; \sum\nolimits_b \text{tenure}_b\ \text{dur}_b\ \text{money}_b \;=\; \sum\nolimits_b \frac{\text{conv}_b}{\text{dur}_b}\ \text{dur}_b\ \text{money}_b \;=\; \sum\nolimits_b \text{conv}_b\ \text{money}_b \;=\; 0$$

$\implies$ **steepener-neutral $\iff$ convexity-neutral**

$\implies$ There is not a bond trade that is both curve-neutral and convexity-positive (for a definition of 'curve' using the centre of the yield-source of a bond). Ye seeking that—and many have—abandon all hope.

Tenure properties and examples

For non-negative cashflows, maturity ≥ tenure ≥ duration. If there are multiple positive payments then the inequalities become strict.

Duration jumps as a coupon is paid. But tenure is continuous: immediately before the coupon, the numerator's t_1^2 term and the denominator's t_1 terms are both almost zero.

The table shows Macaulay duration and tenure for 4% annual-paying par bonds. The last column is the z in $\textit{tenure} = \textit{maturity}^z\ \textit{duration}^{1-z}$; for a perpetual $z \to 0$. For a perpetual on a coupon date, coupon frequency being *freq* times per year and yield r quoted likewise, $\textit{tenure}_\infty = \frac{2}{r} + \frac{1}{\textit{freq}}$ and $\textit{duration}_\infty = \frac{1}{r} + \frac{1}{\textit{freq}}$, both consistent with the table's last row.

Also, instantaneous $\frac{\partial \textit{tenure}}{\partial \textit{time}} = \frac{\textit{tenure}}{\textit{duration}} - 2$.

Macaulay tenures and durations of 4% annual-paying par bonds.

Mat	Ten'r	Dur'n	$\frac{\ln(t/d)}{\ln(m/d)}$
1	1.000	1.000	
2	1.980	1.962	0.4926
3	2.948	2.886	0.5458
5	4.843	4.630	0.5857
7	6.687	6.242	0.6003
10	9.355	8.435	0.6079
12	11.069	9.760	0.6089
15	13.544	11.563	0.6077
20	17.418	14.134	0.6019
25	20.985	16.247	0.5938
30	24.255	17.984	0.5846
40	29.953	20.584	0.5646
50	34.624	22.341	0.5439
70	41.392	24.330	0.5028
100	46.960	25.485	0.4471
400	51.000	26.000	0.2465

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† `https://www.jdawiseman.com/plotting_yields.html` (2008).